Five Year Integrated M.Sc. Examination - 2021
Semester-V
Course MT-3-5-1
(Discrete Mathematics)

Time: Four Hours **Full Marks: 80**

Questions are of value as indicated in the margin
Notations and symbols have their usual meaning

Answer *any eight* questions

1. (a) Define well-formed formula in propositional logic. When does an argument form said to be valid? Construct a formal proof of validity for

$(A \vee B) \to (C \wedge D)$
$(C \vee E) \to (\sim F \wedge G)$
$(F \vee H) \to (A \wedge I)$
$\therefore \sim F.$ [2+4]

(b) Examine validity of the argument

$X \leftrightarrow (Y \to Z)$
$Y \leftrightarrow (\sim X \wedge \sim Z)$
$Z \leftrightarrow (X \vee \sim Y)$
Y
$\therefore X \vee Z.$ [4]

2. Symbolize the following:

(a) If Bengal and Goa both do not win their first games, then Bengal and Goa do not both win their first games. [2]

(b) All that glitters is not gold. [2]

(c) Nothing in the house escaped destruction. [2]

(d) Fruits and vegetables are wholesome and nourishing. [2]

(e) No horse is gentle unless it has been well-trained. [2]

3. Construct a formal proof of validity for the following arguments

(a) $(\exists x)X(x) \to (\forall y)(Y(y) \to Z(y))$
$\therefore (\exists x)(X(x) \wedge Y(x)) \to (\exists y)(X(y) \wedge Z(y)).$ [4]

(b) Any author is successful if and only if he is well-read. All authors are intellectuals. Some authors are successful but not well-read. Therefore, all intellectuals are authors.

4. Examine the validity of the following arguments: [6]

(a) Automobiles and wagons are vehicles. Some automobiles are Fords. Some automobiles are trucks. Some wagons are not vehicles. Therefore, some Fords are trucks.

[5]

(b) $(\forall x)\{(B(x) \to C(x)) \wedge (D(x) \to E(x))$
$(\forall x)[(C(x) \vee E(x)) \to \{F(x) \to (G(x) \to F(x)) \to (B(x) \wedge D(x))\}]$
Therefore, $(\forall x)(B(x) \leftrightarrow D(x)).$ [5]

5. (a) If an $n-$vertex graph G has $(n-1)$ edges and no cycles, then it is connected. [5]

(b) Show that a graph G is a tree if and only if it is loop-free and has exactly one spanning tree. [5]

6. (a) Show that any two longest paths in a connected graph have a vertex in common. [5]

(b) Show that every graph G ($\delta(G) \geq 2$) contains a path of length $\delta(G)$ and a cycle of length at least $\delta(G)+1$. [5]

7. (a) Write an algorithm for the construction of a spanning tree for a connected graph G using Breadth First search. Give an example of your choice. [5]

(b) Show that a connected graph is Euler if and only if all vertices are of even degree. [5]

8. (a) Define planar graph. If all cycles in G have length at least four, then show that $e \leq 2v - 4$, where $v \geq 3$. [5]

(b) Explain vertex colouring of a graph. Define chromatic number. Show that for any simple graph G

$$\chi(G) \leq \Delta(G) + 1.$$

[5]

9. (a) Compute the number of $n-$digit quaternary sequences having an even number of 1s. [5]

(b) Count the number of derangements of the sequence $\{1, 2, \cdots, n\}$. [5]

10. (a) State and prove Dilworth's theorem on counting the maximum size of chain/anti-chain in a large poset. [5]

(b) State and prove Burnside's theorem on counting the number of equivalence classes into which a set is divided by the equivalence relation induced by a permutation group. [5]

11. (a) Integers from 1 to 10 are randomly arranged around a circle. Prove that their must be three consecutive integers whose sum is atleast seventeen. [5]

(b) Given n integers $a_1, a_2, \cdots a_n$, not necessarily distinct, show that there exists integers k and l with $0 \leq k < l \leq n$ such that the sum $a_{k+1} + a_{k+2} + \cdots + a_l$ is a multiple of n. [5]

12. (a) A valid codeword is an $n-$digit decimal number containing an even number of 0s(zeroes). If a_n denote the number of valid codewords of length n, then using generating function technique find an explicit form for a_n. [5]

(b) How many solutions does

$$x_1 + x_2 + x_3 = 11$$

have where $0 \leq x_1 \leq 3$, $0 \leq x_2 \leq 4$ and $0 \leq x_3 \leq 6$? [5]